Gottfried and Bjorken integrals

and

gluon polarization in the proton

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Abstract - We show that the Gottfried and the Bjorken integrals have the same nonperturbative evolution, which is related to the gluon polarization in the proton.

In the present talk we focus our attention on the Gottfried integral and on the Bjorken integral, two quantities which are deduced from data of deep inelastic scattering of leptons on nucleons and which are of great theoretical interest [1,2]. Starting from some assumptions on nonperturbative processes, we show that at low Q^2 these integrals have the same QCD evolution. Furthermore we show that the two integrals are related to the gluon polarization inside the proton, an intriguing quantity to resolving the so-called "spin crisis".

First of all we recall the definitions of the two above mentioned integrals,

$$S_G(Q^2) = \int_0^1 dx[u(x,Q^2) - d(x,Q^2)], \qquad S_B(Q^2) = \int_0^1 dx[\Delta u(x,Q^2) - \Delta d(x,Q^2)], \quad (1)$$

and of other integrals which, as we shall see, result to be related to them, that is

$$\Delta G(Q^2) = \int_0^1 dx \Delta g(x, Q^2), \qquad \Delta\Sigma(Q^2) = \int_0^1 dx \sum_{i=1}^{3} \Delta q_i(x, Q^2), \tag{2}$$

where

$$q_i(\Delta q_i) = q_i^+ \pm q_i^- + \overline{q}_i^+ \pm \overline{q}_i^- \qquad (q_1 = u, \quad q_2 = d, \quad q_3 = s), \qquad \Delta g = g^+ - g^-. \tag{3}$$

According to standard notations q, $\overline{q}$ and g are the quark, antiquark and gluon densities, indices $+$ and $-$ referring to parton helicities. Moreover for the quark spin content $\Delta\Sigma$ in the proton we have adopted a gauge invariant definition [3].

The QCD evolution properties of the Gottfried and of the Bjorken integrals have to be examined in the framework of the hadron structure, which appears different according as we consider masses and magnetic moments, or quantities inferred from deep inelastic scattering, weak and pionic decays. As is well-known, hadronic masses and magnetic moments are described satisfactorily by a nonrelativistic constituent quark model (NRCQM), so that the hadronic states constitute irreducible representations of the $SU(6) \otimes O(3)$ group. On the contrary deep inelastic scattering (DIS) and decays support the current quark picture, with respect to which hadrons appear as complicated mixings of irreducible representations of that group. In particular according to the NRCQM the nucleon belongs to the irreducible representation $\{56, L = 0\}$. Therefore we predict that

$$S_G = S_G^0 = 1; \qquad S_B = S_B^0 = \frac{5}{3}; \qquad \Delta\Sigma = 1. \tag{4}$$

The parton model is not expected to agree with the predictions (4), owing to the above mentioned mixing. Indeed several years ago three sum rules - by Gottfried, Bjorken and Ellis-Jaffe - were formulated about the above mentioned integrals. Among these only the Gottfried sum rule [1], based on the assumption of isoscalar sea, was in accord with (4). The

Bjorken sum rule [2], founded on the parton model and on isospin symmetry, predicted that $S_B \simeq 1.26$; moreover the Ellis-Jaffe sum rule [4], based on the assumptions of unpolarized strange sea ($\Delta s(x) = 0$), and on $SU(3)$ symmetry among hyperon decay constants, led to the prediction $\Delta\Sigma \simeq 0.7$ Surprisingly enough, experiments of a few years ago have not confirmed the Gottfried sum rule and, above all, the Ellis-Jaffe sum rule. The EMC [5] experimental data, combined with $SU(3)$ flavour symmetry, led to the results $\Delta\Sigma = 0.31 \pm 0.07$, $\Delta S < 0$, where ΔS is the spin content of the strange sea. This unexpected result is usually known as the "spin crisis". Furthermore NMC [6] experiments yielded $S_G = 0.72 \pm 0.048$. Only the Bjorken sum rule (based on very general assumptions) is confirmed by experiments [7], provided perturbative QCD [8] corrections are taken into account. In any case we conclude that NRCQM predictions of the quantities S_G, S_B and $\Delta\Sigma$ are always in disagreement with experimental results based on deep inelastic scattering. We interpret these discrepancies as effects of the Q^2-evolution. In particular, as regards the Gottfried and Bjorken integrals, from the data and predictions exposed just above it follows

$$\frac{S_G}{S_G^0} \sim \frac{S_B}{S_B^0} \sim 0.75, \tag{5}$$

where $S^0_{G(B)}$ refer to NRCQM predictions, S_G to DIS measurements and S_B is the r.h.s. of the Bjorken sum rule.

The approximate equality (5) suggests that, at sufficiently low Q^2 (less than $\sim 4\ GeV^2$), S_G and S_B could have the same Q^2-evolution, caused by nonperturbative interactions. In other words we assume an evolution equation similar to the Altarelli-Parisi equation, that is,

$$S(Q^2) = S(Q_0^2) exp[\int_{t_0}^{t} \gamma(t')dt'], \qquad t = log\frac{Q^2}{\mu^2}, \tag{6}$$

where S denotes either the Gottfried or the Bjorken integral and γ a function of t which depends on the nonperturbative interactions. We identify the cause of this evolution with Spontaneous Chiral Symmetry Breaking (SCSB). Let us illustrate in detail the case of the Bjorken integral. To the leading twist this integral is related to the isovector axial current:

$$S_B s_\mu =< P, s|j^3_{5\mu}|P, s > C(Q^2), \qquad s_\mu =< P, s|\gamma_5\gamma_\mu|P, s >, \tag{7}$$

where s is the proton spin and C a real positive function of Q^2, which, according to the Bjorken sum rule, satisfies the condition $\lim_{Q^2\to\infty} C(Q^2) = 1$; that is, an infinite momentum probe "sees" the current quarks as noninteracting and therefore, as illustrated in fig. 1, the axial charge is the same as the one measured in beta-decay. At high but finite Q^2 the current quarks interact perturbatively through gluons and, according to the results by Kodaira et al. [8], cause a reduction of $C(Q^2)$, i. e., of the effective axial charge "seen" by virtual photons. However terms of order α_s^2 tend to increase the effective charge. When Q^2 becomes sufficiently small, nonperturbative processes have to be taken into account and increasing effects prevail over reduction ones; in particular SCSB converts current quarks into constituent quarks, so as to increase $C(Q^2)$, therefore S_B varies from ~ 1.26 to ~ 1.67, according to the NRCQM.

Now we show that according to the chiral model by Ball and Forte [9] (named BF in the following) the Gottfried and the Bjorken integrals have the same evolution for small Q^2. The model is based on nonperturbative interactions between quarks and pseudoscalar mesons. These interactions, pictured in fig. 2, are dominant at sufficiently low Q^2 and cause an evolution of the quark densities inside the proton, similarly to the Altarelli-Parisi perturbative splitting functions. In the BF model the function γ, which appears in the first eq. (6), is given by $\gamma = \frac{d}{dt}(\frac{1}{2}\sigma_{\gamma^*\pi^0} + \frac{1}{3}\sigma_{\gamma^*\eta} + \frac{1}{6}\sigma_{\gamma^*\eta'} - \sigma_{\gamma^*\pi^+})$, where the σ are the cross

sections for the processes $\gamma^* q \longrightarrow q' M$, $(M = \pi, \eta, \eta')$ and t is the evolution parameter defined by the second eq. (6). The axial anomaly produces differences among the meson masses, which in turn cause the coefficient γ not to vanish. Indeed, the largest contribution to γ comes from the graph containing the pion. BF show that this coefficient is particularly important for not too small Q^2, while for Q^2 comparable with the mass squared of the pion it is negligibly small. Then the chiral approximation may be adopted. In the rest frame of a pion the quark and the antiquark have equal helicities; since for a massless quark the helicity is independent of the reference frame, the helicity of the final quark in the splitting function is equal to that of the initial quark (see fig. 2). This implies, similarly to the Altarelli-Parisi splitting functions, that polarized and unpolarized nonsinglet structure functions have the same evolution. So the approximate equality (5) follows from the BF model.

Now we examine the question from a complementary viewpoint. The components of the hadronic tensor may be either symmetric or antisymmetric, according as we consider unpolarized deep inelastic scattering cross section or asymmetry with polarized target and beam. Furthermore we distinguish between the isovector and the isoscalar components of the hadronic tensor. In the present talk we are interested only in the isovector components: the (anti-)symmetric component is proportional to $S_G(S_B)$. Fig. 1 represents the antisymmetric isovector component of the hadronic tensor, that is, the forward Compton scattering with the exchange of an isovector pseudovector object. This exchange is described by a meson Regge trajectory. Indeed, since the antisymmetric isovector component of the hadronic tensor is proportional to the isovector axial current, the Goldberger-Trieman relation implies the dominance of the pion Regge trajectory. In principle also the symmetric part of the isovector hadronic tensor could be dominated by a Regge trajectory with the right quantum numbers,

i. e. by the exchange of a scalar isovector object. However SCSB removes the degeneracy between the pion and the ρ-meson Regge trajectories, so that no simple reggeized meson exchange dominates the isovector component of unpolarized forward Compton scattering. A more complex object has to be hypothesized. In particular, as illustrated in fig. 3, such an object could be constituted by the Lorentz invariant product of the isovector axial current times the Chern-Simons current, defined as $k_\mu = \frac{\alpha_s}{2\pi}\epsilon_{\mu\nu\lambda\sigma}\, Tr[A^\nu(G^{\lambda\sigma} - \frac{2}{3}A^\lambda A^\sigma)]$, where A is the gluon field and G the QCD strength tensor field. Therefore we set

$$S_G = FC(Q^2) < P|j^3_{5\mu}k^\mu|P >, \tag{8}$$

where the constant F, Q^2-independent, has been chosen in such a way that $\lim_{Q^2\to 0} S_G = 1$. Indeed the operator $j^3_{5\mu}k^\mu$ has the right quantum numbers for being exchanged in the isovector symmetric hadronic tensor, furthermore it does dominate in the forward Compton scattering, nor do we know an equally simple operator which may have a comparable weight in such a process.

In order to draw the consequences of assumption (8), we recall some important properties of the Chern-Simons current, which plays an essential role in two important effects, like the η'-particle mass shift with respect to the pseudoscalar meson octet [10] (fig. 4) and the "spin crisis" [3].

In connection with the latter effect, we precise that the forward SU(3)-singlet axial form factor $\Delta\Sigma$ consists in two addends, called respectively connected ($\Delta\Sigma'$) and disconnected ($-\Delta\Gamma$) insertions [3] (fig. 5), i. e.,

$$\Delta\Sigma = \Delta\Sigma' - \Delta\Gamma \tag{9}$$

Furthermore, $\Delta\Sigma$ corresponds to a gauge-invariant, Q^2-dependent definition of the quark

spin content inside the proton, that we have adopted before (see eq. (2)). On the contrary $\Delta\Sigma'$ defines the same quantity in a gauge-variant, Q^2-independent way. $\Delta\Gamma$, which represents the spin screening contribution of the sea quarks, caused by photon-gluon scattering (fig. 6), is related to the Chern-Simons current:

$$< P, s|k_\mu|P, s >= c_0 s_\mu \Delta\Gamma, \tag{10}$$

where c_0 is a Q^2-independent positive constant. Taking into account equations (7), (8) and (10) yields

$$S_G = -c_0 F S_B \Delta\Gamma, \tag{11}$$

which implies that the product F $\Delta\Gamma$ must be negative. If, as we are going to show, $\Delta\Gamma$ is very slowly Q^2-dependent, eq. (11) implies that S_G and S_B have approximately the same Q^2-dependence, which proves the approximate equality (5).

For sufficiently large Q^2 α_s is small and $\Delta\Gamma$ evolves like α_s^2. For smaller Q^2 (of order 1 GeV^2) the perturbative evolution no longer holds true and only nonperturbative models, like the BF model, can be invoked. In the BF model the evolutions of the quantities $\Delta\Sigma$ and ΔG are controlled by splitting functions in which only the η' particle is involved (fig. 7), therefore the QCD evolution of such quantities is negligible in comparison with that of S_B (see also ref. [11]). Since $\Delta\Sigma'$ is Q^2-independent, from eq. (9) it follows that also $\Delta\Gamma$ has a negligible evolution.

We conclude observing that the quantity ΔG is crucial in resolving the "spin crisis". Presently three possible scenarios may be assumed:

i) $\Delta\Sigma' \simeq 0.7$, according to the Ellis-Jaffe sum rule, which implies $\Delta G \simeq 2$;

ii) $\Delta\Sigma' \simeq \Delta\Sigma \simeq 0$, according to chiral models (e. g. [12]), which implies a very small value of ΔG (≤ 0.15);

iii) $\Delta G < 0$, according to considerations about interactions between constituent quarks and gluons [13]. In this case the constant F, which appears in eqs. (8) and (11), is positive, while in cases i) and ii) it is negative.

Several experiments have been suggested for distiguishing among the above mentioned scenarios [14]. Furthermore data form experiment FNAL E581 [15], although affected by large uncertainties, indicate a small, positive ΔG.

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FIGURE CAPTIONS

[Fig.1] - The antisymmetric isovector component of the hadronic tensor compared with the amplitude of beta-decay

[Fig.2] - Splitting functions: a) Altarelli-Parisi graphs; b) quark-meson interactions. Arrows indicate helicities.

[Fig.3] - The symmetric isovector component of the hadronic tensor according to our assumption.

[Fig.4] - Coupling scheme proposed by 'tHooft [10] for explaining the mass shift of the η' particle.

[Fig.5] - The forward SU(3)-singlet axial form factor: a) connected insertion; b) disconnected insertion

[Fig.6] - Quark spin screening produced by photon-gluon scattering.

[Fig.7] - Splitting function for evolution of gluon densities and of SU(3)-singlet combination of quark densities.